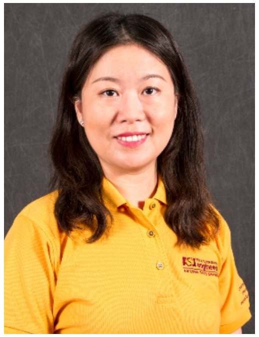
On the Connectivity of Multi-layered Networks: Models, Measures and Optimal Control

Presented by Chen Chen


Chen Chen


Jingrui He


Nadya Bliss

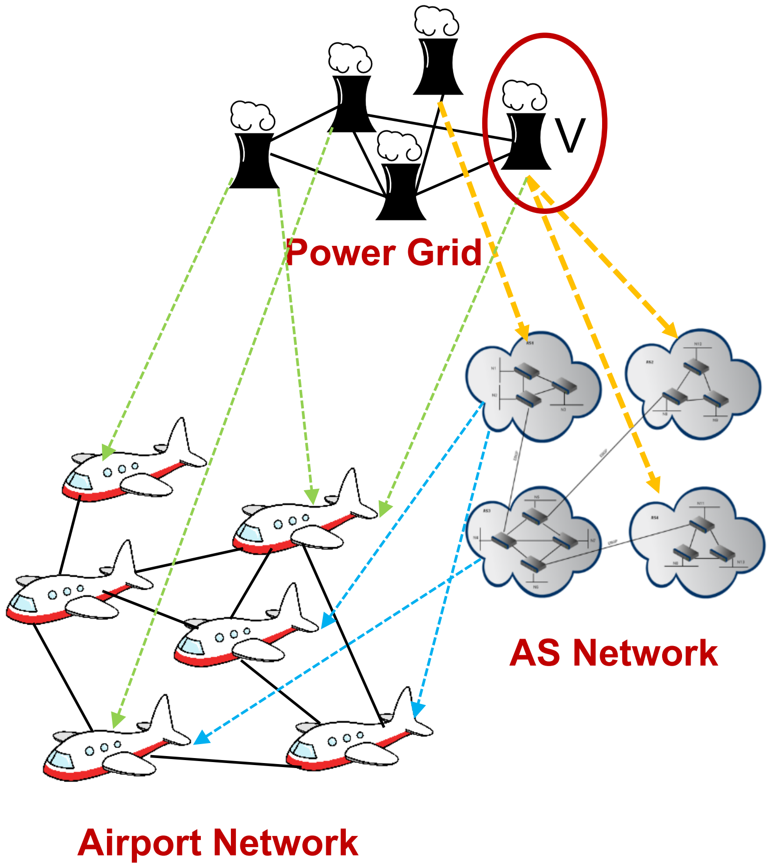

Hanghang Tong

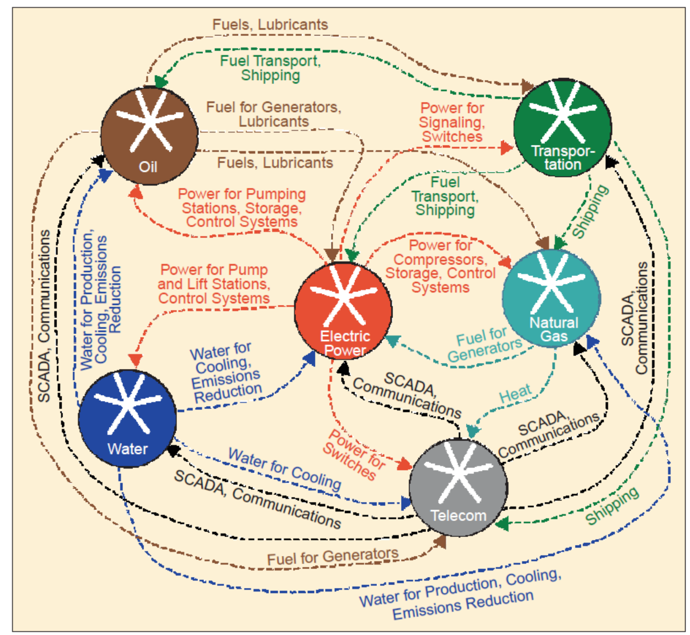
DATA Lab

Arizona State University

Multi-layered Networks are Prevalent!

"Learn how to see. Realize that everything is connected to everything else." ---Leonardo da Vinci



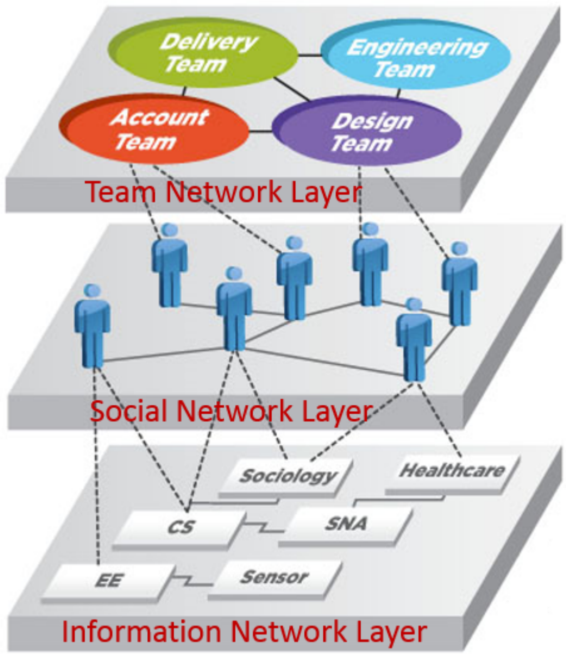


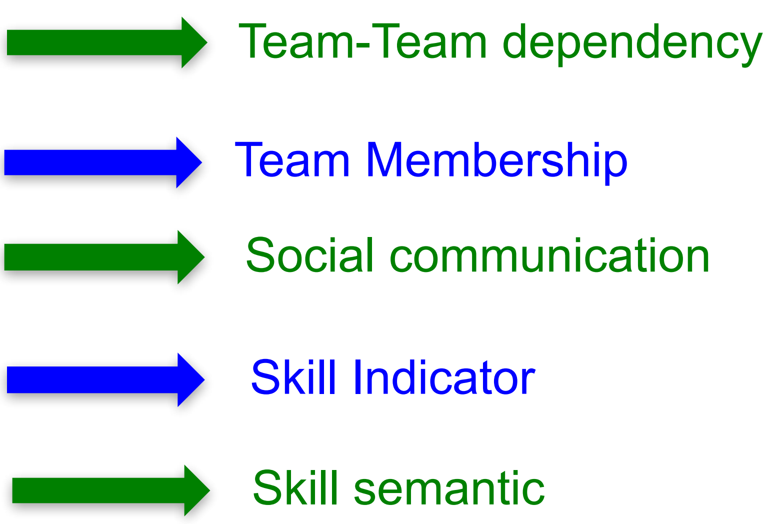
- Infrastructure Interdependencies

This Paper: (Q1) Models; (Q2) Measures and (Q3) Control

Q1: Model Multi-layered Networks

- Obs.: Rich Connectivity Structure
  - An Example of Collaboration Networks





- Q: How to Simultaneously represent within-layer and cross-layer connectivity?

Q2: Unify Connectivity Measures

- Obs.: Many Different Connectivity Measures
  - Path Capacity (Epidemic Threshold)

$$C_{epid} \sim {}_{<A,1>}$$

  - Loop Capacity (Natural Connectivity)

$$C_{natr} \sim {}_{i=1}^{n} e^{<A,i>}$$

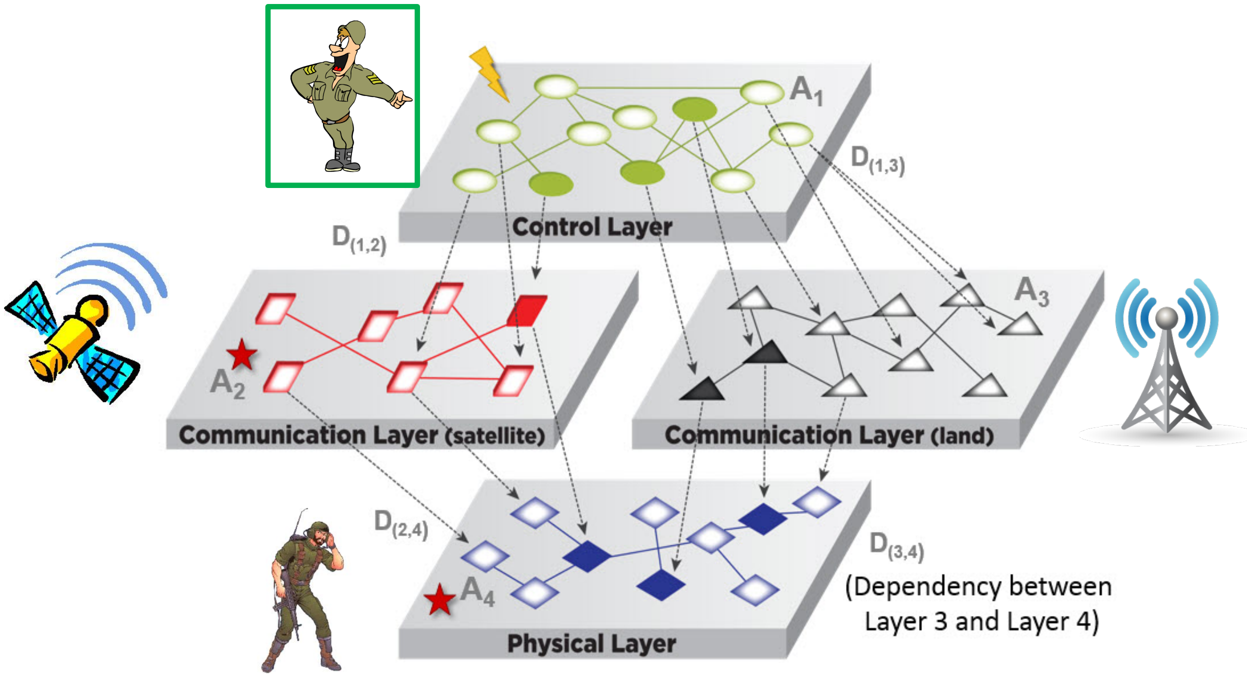
  - Triangle Capacity (Social Balance Theory)

$$C_{tri} \sim {}_{i=1}^{n} {}^{3}_{<A,i>}$$

- Q: Which One to Use? Any Commonality?

Q3: Control Connectivity Control

- Obs.: (1) Limited Access + (2) Butterfly Effect

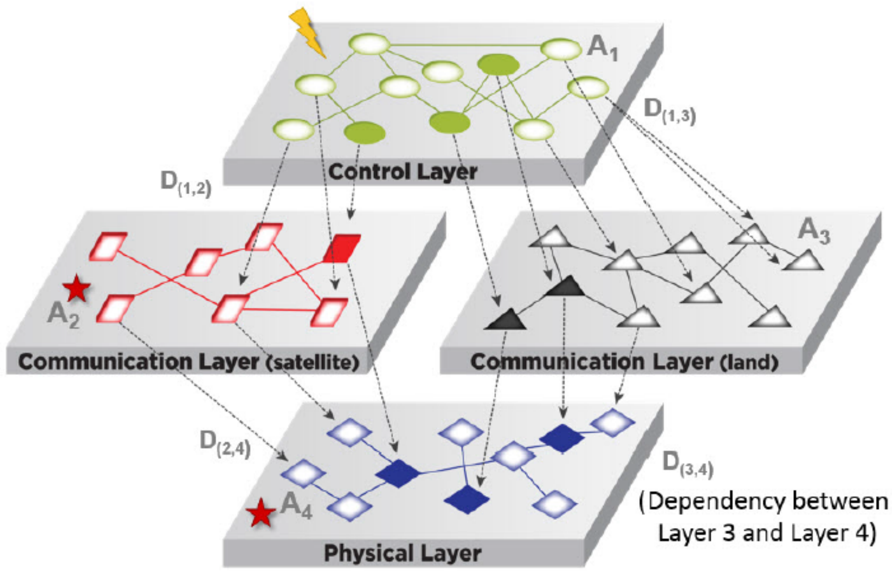


- Q: How to optimally operate the ***control layer***, to minimize the connectivity of the ***target layer(s)***?

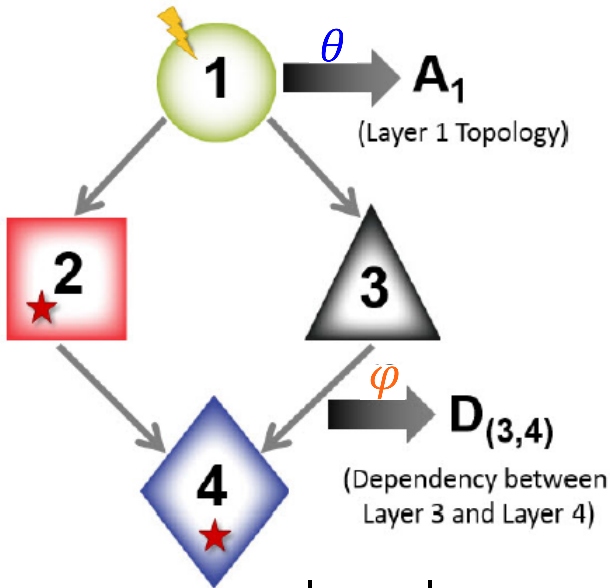
Roadmap

- Motivation
- Q1: **Mu**lti-**la**yered **N**etwork (MULAN) Models
- Q2: Unification of Connectivity Measures
- Q3: Optimal Connectivity Control
- Evaluations
- Conclusions

Q1: Multi-layered Networks Models



A four-layered network



layer-layer dependency network

- **A Quintuple:** $\Gamma = <G, \boldsymbol{A}, \boldsymbol{D}, \theta, \varphi>$
  - G: layer-layer dependence network
  - $\boldsymbol{A}$: intra-layer connectivity
  - $\boldsymbol{D}$: cross-layer dependence
  - θ: function $V_G \rightarrow \boldsymbol{A}$
  - φ: function $E_G \rightarrow \boldsymbol{D}$

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Q2: Connectivity Unification
(SUBLINE Family)

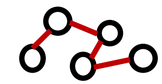
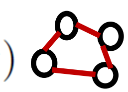
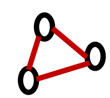
- **Key Idea**: graph connectivity as an **aggregation** over the subgraph connectivity:

$$C(\mathbf{A}) = \sum_{\pi \subseteq \mathbf{A}} f(\pi)$$

  - A: adjacency matrix of the graph
  - π: a non-empty subgraph in A
  - $f(\pi)$: connectivity of the subgraph π
  - $C(A)$: connectivity of graph A

SUBLINE Family

- **Key Idea:** $\underline{C(\mathbf{A})} = \sum_{\underline{\pi \subseteq \mathbf{A}}} \underline{f(\pi)}$

- **Examples**
  - Path Capacity: $f(\pi) = \begin{cases} \beta^{len(\pi)} & \text{if } \pi \text{ is a valid path of length } len(\pi) \\ 0 & \text{otherwise.} \end{cases}$ 
  - Loop Capacity: $f(\pi) = \begin{cases} 1/len(\pi)! & \text{if } \pi \text{ is a valid loop of length } len(\pi) \\ 0 & \text{otherwise.} \end{cases}$ 
  - Triangle Capacity: $f(\pi) = \begin{cases} 1 & \text{if } \pi \text{ is a triangle} \\ 0 & \text{otherwise.} \end{cases}$ 
  - …

All of them fall into the subline family!

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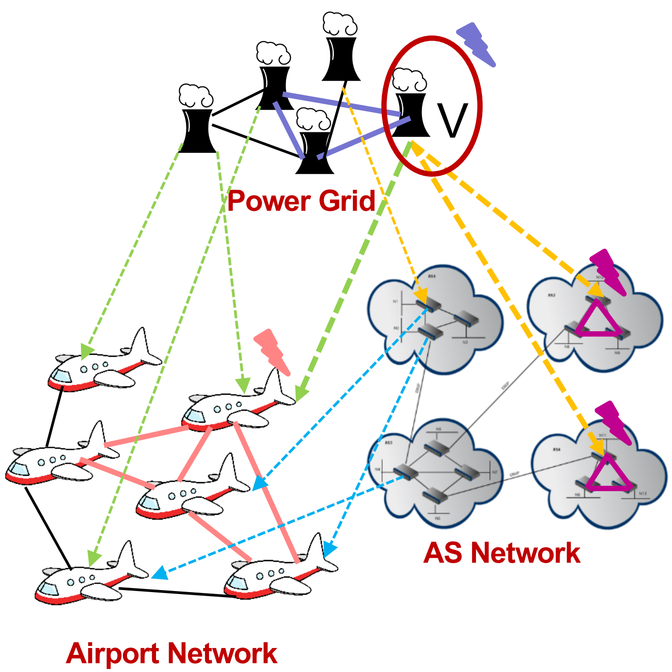
Connectivity Control in MULAN

- Define $\mathbf{I}(S_i) = \sum_{i=1}^{g} \alpha_j (C(A_j) - C(A_j \backslash S_{i \to j}))$
  - Example: $C =$ Triangle Capacity

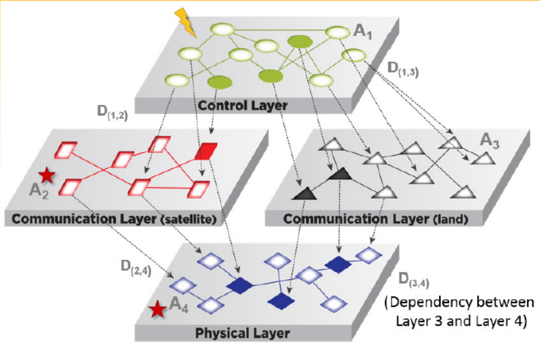
$\mathbf{I}$(V) ~ #Triangles in which A & its dependencies participate



$\mathbf{I}(V) \sim 1 + 2 + 2$

Power Grid — AS Network — Airport Network

Optimal Control in MULAN



- Goal
  - Find an optimal node set in the control layer to maximize its impact on the target layers
- Theorem
  - The SUBLINE connectivity control problem enjoys the **diminishing returns property**
- Solutions
  - Greedy algorithm → linear, near optimal

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Evaluations

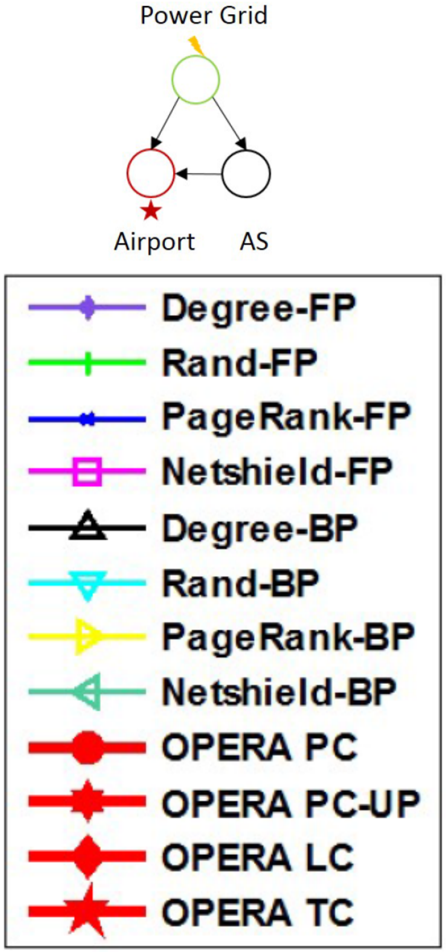
- Datasets
  - **MULTIAS**: Multi-layered Internet topologies at AS level
  - **INFRNET**: Critical infrastructure networks
  - **SOCINNET**: Social-information collaboration networks

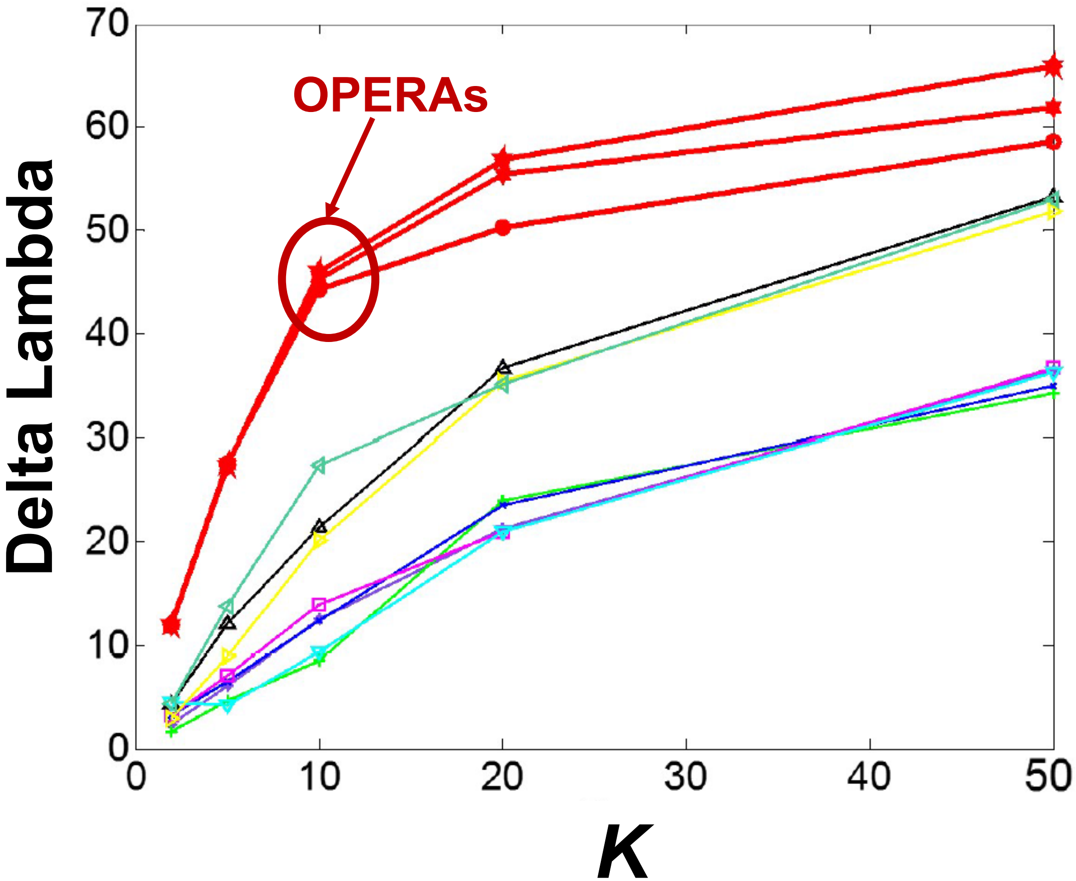
Data Sets	Application Domains	# of Layers	# of Nodes	# of Links
D1	MULTIAS	2∼4	5,929∼24,539	11,183∼50,778
D2	INFRANET	3	19,235	46,926
D3	SOCINNET	2	63,501∼124,445	13,097∼211,776

- Evaluation objectives
  - **Effectiveness**: OPERAs outperform other heuristics
  - **Efficiency**: OPERAs scale linearly w.r.t graph size

Effectiveness

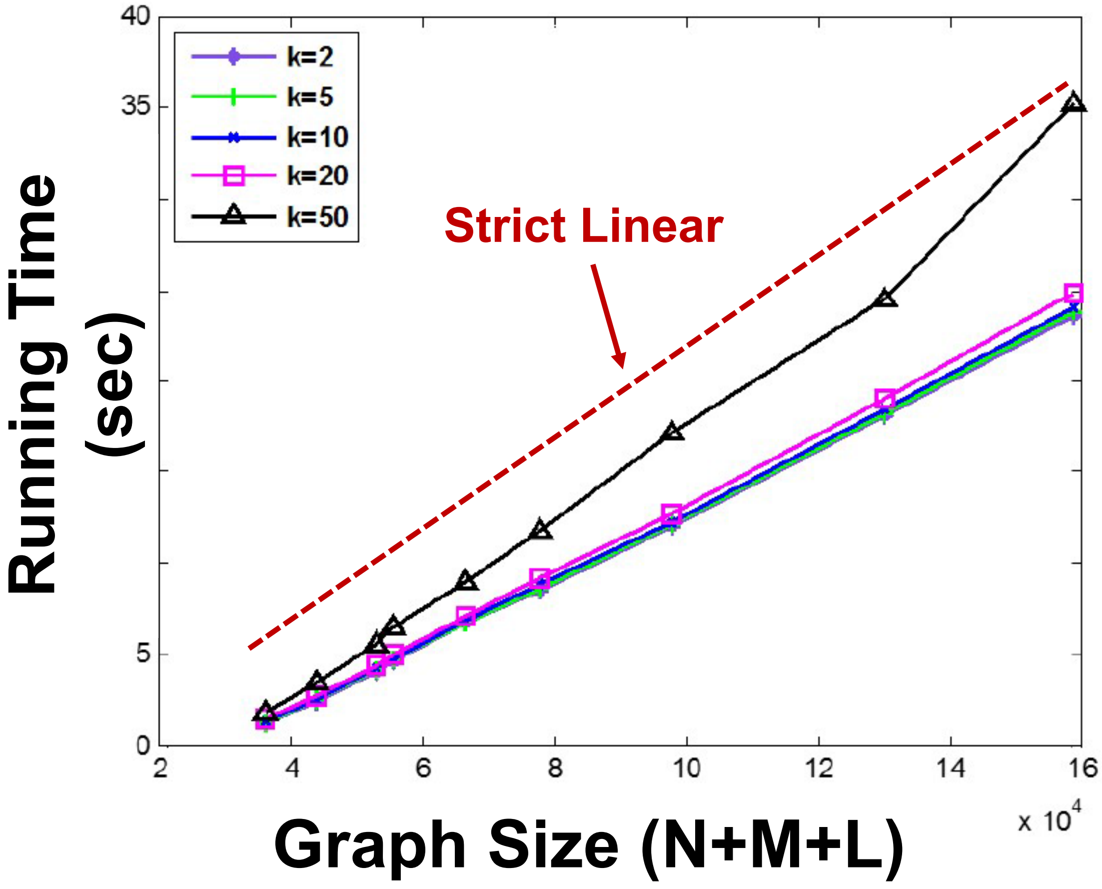
- INFRANET





Scalability

- OPERA PC



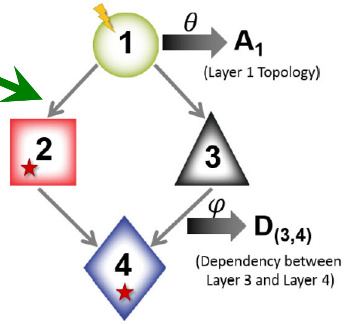
Conclusions

- Connectivity on Multi-Layered Networks
  - Q1: Model of Multi-layered Networks
  - A1: MULAN ($\Gamma = <G, \boldsymbol{A}, \boldsymbol{D}, \theta, \varphi>$)

  - Q2: Unification of Connectivity Measures
  - A2: SUBLINE family

$$C(\mathbf{A}) = \sum_{\pi \subseteq \mathbf{A}} f(\pi)$$

  - Q3: Optimal Connectivity Control
  - A3:A generic solution (linear, near-optimal)



- More in paper
  - Proof of diminishing returns property
  - More experimental results